

What you'll Learn About

- Two Dimensional Vectors/Vector Operations/Unit Vectors
- Direction Angle/Applications of Vectors

$$\overrightarrow{2QR} + \overrightarrow{PS}$$

Vector:

★ Direction

★ Length

Magnitude
↳ Pythagorean theorem



Find the component form and magnitude of the vector where $P = (-3, 4)$, $Q = (-5, 2)$, $R = (-1, 3)$ and $S = (4, 7)$

A) \vec{PQ}
Initial Point → Terminal Point

$$(-5, 2) \quad (-3, 4)$$

$$\langle -5 - (-3), 2 - 4 \rangle$$

$$\langle -2, -2 \rangle$$

$$\text{mag} \rightarrow a^2 + b^2 = c^2$$

$$\text{mag} = \sqrt{(-2)^2 + (-2)^2}$$

$$= \sqrt{4 + 4}$$

$$= \sqrt{8}$$

C) $3\vec{QS}$

$$3 \langle 4 - (-5), 7 - 2 \rangle$$

$$3 \langle 9, 5 \rangle$$

$$\langle 27, 15 \rangle$$

$$\text{mag} = \sqrt{(27)^2 + (15)^2}$$

$$= \sqrt{954}$$

B) \vec{RS}

Initial Point → Terminal Point
 $R(-1, 3)$ $S(4, 7)$

$$\langle 4 - (-1), 7 - 3 \rangle$$

$$\langle 5, 4 \rangle$$

$$\begin{aligned}\text{mag} &= \sqrt{(5)^2 + (4)^2} \\ &= \sqrt{25 + 16} \\ &= \sqrt{41}\end{aligned}$$

D) $2\vec{QR} + \vec{PS}$

$$2 \langle -1 - (-5), 3 - 2 \rangle + \langle 4 - (-1), 7 - 4 \rangle$$

$$2 \langle 4, 1 \rangle + \langle 7, 3 \rangle$$

$$\langle 8, 2 \rangle + \langle 7, 3 \rangle$$

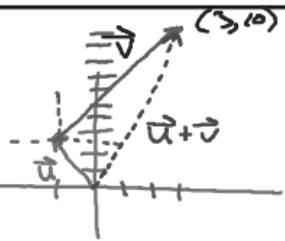
$$\langle 15, 5 \rangle$$

$$\sqrt{15^2 + 5^2}$$

$$\sqrt{225 + 25}$$

$$\sqrt{250}$$

tail-to-head



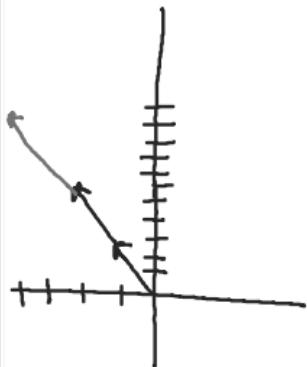
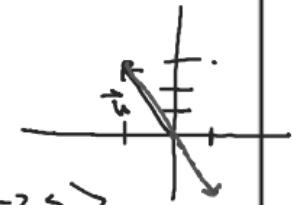
Let $\mathbf{u} = \langle -1, 3 \rangle$, $\mathbf{v} = \langle 4, 7 \rangle$ and $\mathbf{w} = \langle -2, 5 \rangle$. Find the component form of the vector.

A) $\mathbf{u} + \mathbf{v}$

$$\begin{aligned}\langle -1, 3 \rangle + \langle 4, 7 \rangle \\ \langle -1+4, 3+7 \rangle \\ \langle 3, 10 \rangle\end{aligned}$$

B) $\mathbf{u} - \mathbf{w}$

$$\begin{aligned}\mathbf{u} + -\mathbf{w} \\ \langle -1, 3 \rangle - \langle -2, 5 \rangle \\ \langle -1 - (-2), 3 - 5 \rangle \\ \langle 1, -2 \rangle\end{aligned}$$



$$\vec{u} = \langle -1, 3 \rangle \quad \vec{v} = \langle 4, 7 \rangle \quad \vec{w} = \langle -2, 5 \rangle$$

C) $2\mathbf{u} + 3\mathbf{w}$

$$\begin{aligned}2\langle -1, 3 \rangle + 3\langle -2, 5 \rangle \\ \langle 2, 6 \rangle + \langle -6, 15 \rangle \\ \langle -8, 21 \rangle\end{aligned}$$

D) $-2\mathbf{u} - 3\mathbf{v}$

$$\begin{aligned}-2\langle -1, 3 \rangle - 3\langle 4, 7 \rangle \\ \langle 2, -6 \rangle + \langle -12, -21 \rangle \\ \langle -10, -27 \rangle\end{aligned}$$



Unit Vector
Same direction
as original vector
with a magnitude
of 1.

Find unit Vector
original vector
divided by the
magnitude.

Find a unit vector in the direction of the given vector. Write your answer in component form and as a linear combination of the standard unit vectors \mathbf{i} and \mathbf{j}

A) $\mathbf{u} = \langle -1, 3 \rangle$

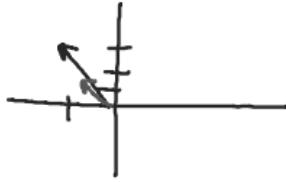
$$\text{mag} = \sqrt{(-1)^2 + (3)^2}$$

$$= \sqrt{10}$$

Unit Vector

$$\left\langle \frac{-1}{\sqrt{10}}, \frac{3}{\sqrt{10}} \right\rangle$$

$$-\frac{1}{\sqrt{10}}\mathbf{i} + \frac{3}{\sqrt{10}}\mathbf{j}$$



$\mathbf{w} = \langle -2, 5 \rangle$

$$\text{mag} = \sqrt{(-2)^2 + 5^2}$$

$$= \sqrt{4+25}$$

$$= \sqrt{29}$$

$$\left\langle \frac{-2}{\sqrt{29}}, \frac{5}{\sqrt{29}} \right\rangle$$

$$-\frac{2}{\sqrt{29}}\mathbf{i} + \frac{5}{\sqrt{29}}\mathbf{j}$$

B) $\mathbf{v} = \langle 4, 7 \rangle$

$$\text{mag} = \sqrt{4^2 + 7^2}$$

$$= \sqrt{16+49}$$

$$= \sqrt{65}$$

Unit Vector

$$\left\langle \frac{4}{\sqrt{65}}, \frac{7}{\sqrt{65}} \right\rangle$$

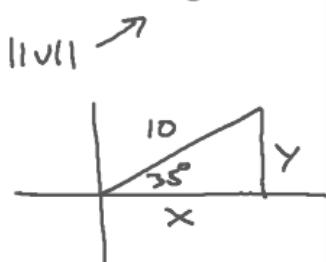
$$\frac{4}{\sqrt{65}}\mathbf{i} + \frac{7}{\sqrt{65}}\mathbf{j}$$

$$= \sqrt{\left(\frac{-2}{\sqrt{29}}\right)^2 + \left(\frac{5}{\sqrt{29}}\right)^2}$$

$$\sqrt{\frac{4}{29} + \frac{25}{29}} = \sqrt{\frac{29}{29}}$$

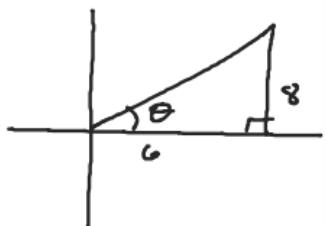
$$= 1$$

$$|v| \rightarrow \text{mag}$$



$$X = \text{mag} \cos \theta$$

$$Y = \text{mag} \sin \theta$$



$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

Find the component form of the vector v with the given magnitude and angle.

A) $v = |10| \theta = 35^\circ$

$$\cos 35^\circ = \frac{x}{10}$$

$$x = 10 \cos 35^\circ \\ = 8.19$$

$$\sin 35^\circ = \frac{y}{10}$$

$$y = 10 \sin 35^\circ \\ = 5.74$$

$$\langle 10 \cos 35^\circ, 10 \sin 35^\circ \rangle$$

$$\langle 8.19, 5.74 \rangle$$

B) $v = |20| \theta = 135^\circ$

$$\langle 20 \cos 135^\circ, 20 \sin 135^\circ \rangle$$

$$\langle -10\sqrt{2}, 10\sqrt{2} \rangle$$

$$\langle -14.14, 14.14 \rangle$$

Find the magnitude and direction angle of the vector.

A) $\langle 6, 8 \rangle$

$$\text{mag} = \sqrt{6^2 + 8^2}$$

$$= \sqrt{100}$$

$$= 10$$

$$\theta = \tan^{-1}\left(\frac{8}{6}\right)$$

$$= 53.13^\circ$$

B) $6i - 8j$

$$\langle 6, -8 \rangle$$

$$\text{mag} = 10$$

$$\theta = -53.13$$

$$360 - 53.13 \\ 306.87^\circ$$

C) $10(\cos 235^\circ i + \sin 235^\circ j)$

$$10 \langle \cos 235^\circ, \sin 235^\circ \rangle$$

$$\langle 10 \cos 235^\circ, 10 \sin 235^\circ \rangle$$

$$\text{mag} = 10$$

$$\theta = 235$$

Navigation

A) An airplane is flying on a bearing of 135° at 435 mph. Find the component form of the velocity of the airplane.

$$\vec{P} = \langle 435 \cos 135^\circ, 435 \sin 135^\circ \rangle$$

$$\langle -307.51, 307.51 \rangle$$



B) An airplane is flying on a compass heading(bearing) of 315° at 300 mph. A wind is blowing with the bearing 220° at 30 mph.

- Find the component form of the velocity of the airplane.

$$\vec{P} = \langle 300 \cos 315^\circ, 300 \sin 315^\circ \rangle$$

- Find the component form of the velocity of the wind.

$$\vec{w} = \langle 30 \cos 220^\circ, 30 \sin 220^\circ \rangle$$

- Find the actual ground speed and direction of the airplane

$$\vec{P} + \vec{w} = \langle 300 \cos 315^\circ + 30 \cos 220^\circ, 300 \sin 315^\circ + 30 \sin 220^\circ \rangle$$

$$\langle 189.15, -231.42 \rangle$$

Speed of plane is magnitude

$$\sqrt{(189.15)^2 + (-231.42)^2}$$

298.886 mph

$$\theta = \tan^{-1} \left(\frac{-231.42}{189.15} \right) = -50.75^\circ$$

$$= 309.27^\circ$$

C) A ship is heading due south at 15 mph. The current is flowing northwest at 3 mph. Find the actual bearing and speed of the ship.

Shooting a basketball: A basketball is shot at an angle 65° with an initial speed of 12m/sec.

a. Find the component form of the initial velocity.

b. Give an interpretation of the horizontal and vertical components of the velocity.

Combining Forces: A force of 40 lbs acts on an object at angle of 20° . A second force of 65 pounds acts on the object at an angle if -25° . Find the direction and magnitude of the resultant force.

